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The Mechanical Behavior of Microporous Polyurethane Foams

R. E. WHITTAKER, *Shoe and Allied Trades Research Association, Kettering, Northants, England*

Synopsis

The mechanical behavior of microporous polyurethane foams used in poromeric materials can be described by use of a model comprising of struts of square cross section arranged in a cubical lattice. The model was initially proposed by Gent and Thomas to describe the properties exhibited by natural rubber latex foams. The microporous polyurethane foams used in poromerics are in general much stronger than natural rubber foams, and it has been found that their tear and tensile properties are dependent on the size of the largest pore, which can be up to 20 times greater in diameter than the average pore size. The behavior of the polyurethane foam in compression can be satisfactorily described by use of this cubical model and shape factor theories from polymer engineering.

INTRODUCTION

The study of the mechanical properties of cellular or foamed polymers as distinct from solid materials was started in the late 1920's with the development of blown or expanded rubber. A number of early papers[1–3] discussed the physical properties such as density, hardness, tensile, hysteresis, damping, cell size, and insulation properties of these materials.

Latex foam rubber was developed in the early 1930's, and a number of investigations have been undertaken[4–7] into the tensile and compression properties of these materials. Most authors showed that the load–extension curves of foamed materials were sigmoidal, but little theoretical work analyzing such deformations was reported although an extensive analysis[8–10] of the elastic properties of cork was made in 1946 and showed that the sigmoidal load–compression curves obtained with cork could be interpreted on the basis of collapsing of cell walls.

It was not until 1959 that a theory to describe the mechanical properties of foamed elastic materials such as modulus, compression, tear, and tensile was developed by Gent and Thomas.[11–13] This theory has now been developed further to describe viscoelastic[14] and permeability[15] properties of open-cell foamed materials and elastic behavior of closed-cell materials[13] and has been successfully applied to measurements on natural rubber foams.

Little work, however, has been reported on the application of a theoretical model to the mechanical behavior of polyurethane foams. The advent of poromerics[16,17] into the footwear industry has necessitated some investiga-

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tion into the strength and mechanical properties of microporous polyurethane foams, and it has been found that these materials are extremely strong when compared with vulcanized synthetic or natural solid rubber.

This paper discusses the modulus, compression, tear, and tensile properties of polyurethane foams used in poromeric materials and relates these measurements to the theoretical model proposed by Gent and Thomas and also to other established theories from rubber elasticity and polymer engineering.

EXPERIMENTAL

Samples of polyurethane foams were obtained from two commercially available poromerics: foam 1 was approximately 0.17 cm thick, while foam 2 was only 0.014 cm thick. It was necessary in the analysis of the results to obtain certain measurements on the solid polyurethane used in the foam. Unfortunately, it was not possible to obtain the solid polymer direct, and hence it was necessary to dissolve the foam in a suitable solvent and to cast the solid material. The solvent was then drawn off under heat. The densities of the foam and solid were measured in both cases. Tensile, tear, and compression data on the materials were obtained by use of an Instron tensile testing machine using suitable jaws and attachments for each particular experiment.

The type of cell structure found in the polyurethane foams can be seen in the stereoscan[18] photomicrograph shown in Figure 1 above. The cells are reasonably spherical, with the average diameter about 10^{-3} cm, and can clearly be seen to be interconnecting.

THEORETICAL MODEL

The model proposed by Gent and Thomas[13] for a foamed material is shown in Figure 2; it consists of thin threads of unstrained length l_0 and cross-sectional area D^2 joined together to form a cubical lattice. The intersections of cubical regions of volume D^3 are assumed to be essentially undeformable.

A fractional extension of the foam by an amount e' parallel to one set of threads is therefore associated with a larger extension e of the threads themselves, as follows:

$$\frac{e}{e'} = \frac{l_0 + D}{l_0} = 1 + \beta. \tag{1}$$

The threads in the model occupy, for any cross section perpendicular to one set of threads, a fractional area of the total given by

$$\frac{D^2}{(D + l_0)^2} = \frac{\beta^2}{(1 + \beta)^2}. \tag{2}$$

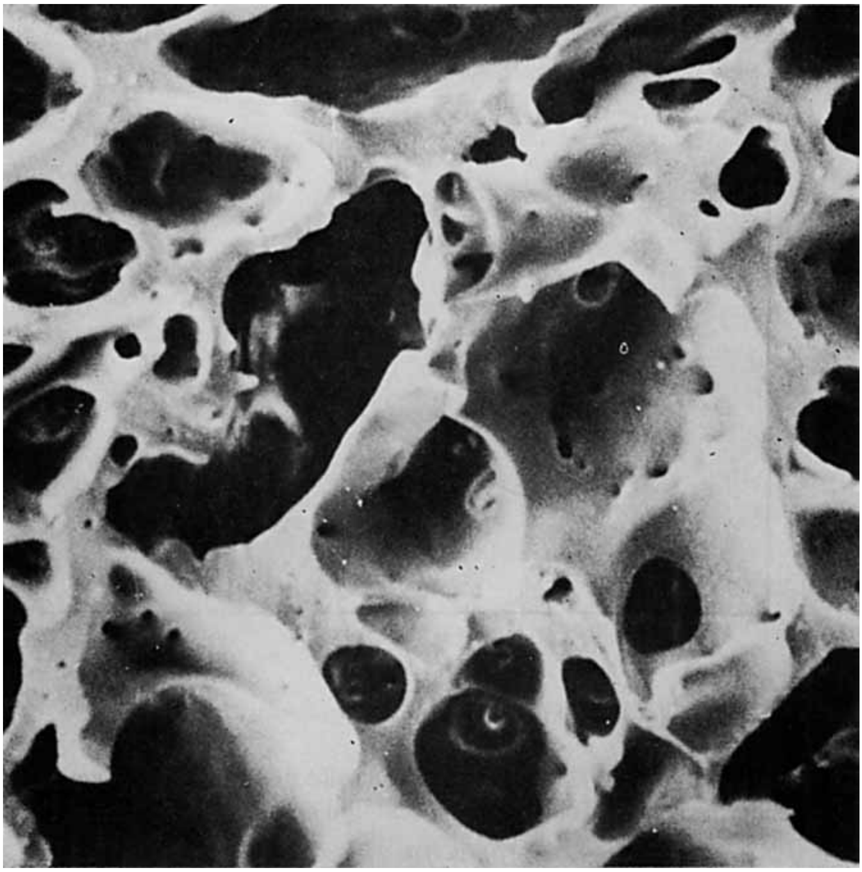

Fig. 1. Scanning electron microscope photograph of polyurethane foam showing type of cell structure. Magnification 3,2000X.

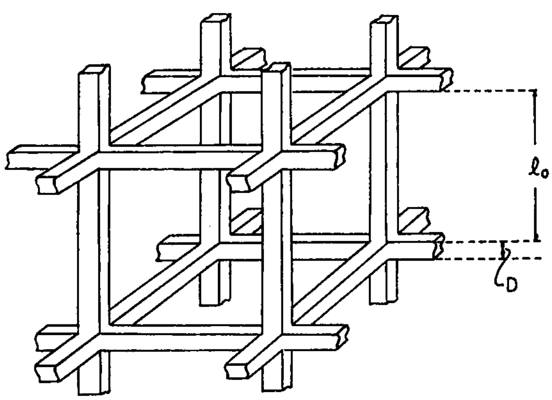

Fig. 2. Simple model of foamed material. After Gent and Thomas.[13]

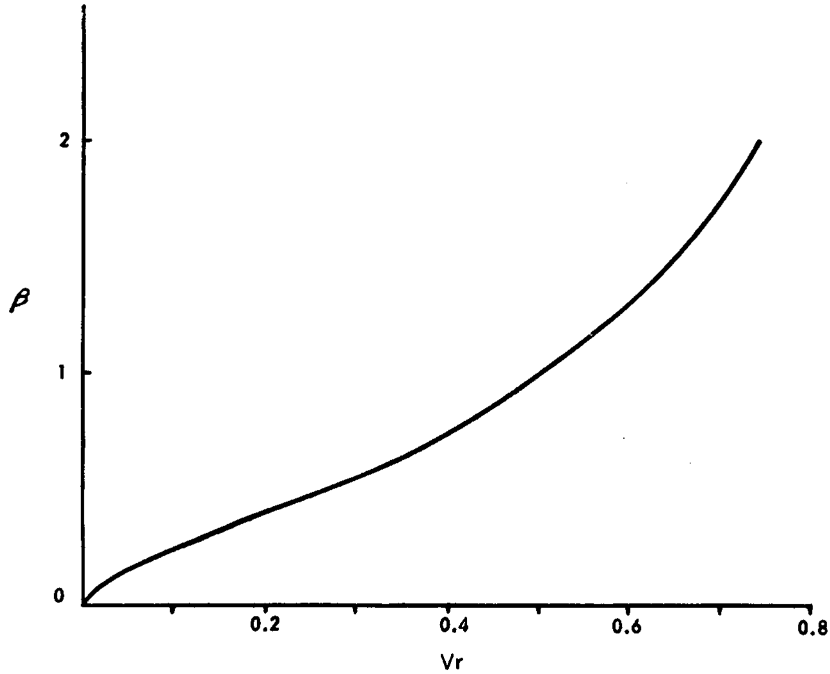

Fig. 3. Variation of parameter β with foam density, from eq. (3).

The fractional volume V_r occupied by the solid material can be evaluated by considering a cube of side $(D + l_0)$ centered on one intersection, so that

$$V_r = \frac{3D^2l_0 + D^3}{(D + l_0)^3} = \frac{3\beta^2 + \beta^3}{(1 + \beta)^3}. \tag{3}$$

The parameter β therefore gives a direct measure of the foam density, and the relationship is shown graphically in Figure 3.

MODULUS

The tensile stress–strain curves of foam 1 and the corresponding solid material are shown in Figure 4. The tensile stress for the foam is based on the cross-sectional area of rubber, including holes. The results for a typical unfilled solid natural rubber vulcanizate from previous studies[19,20] are also shown in Figure 4 for comparison; and it can be seen that the initial modulus of the polyurethane foam is higher, although the actual tensile strength is lower than the NR vulcanizate. The modulus of the solid polyurethane is extremely high when compared to the corresponding foam, and its tensile strength is considerably in excess of that found in the natural rubber vulcanizate. The initial linear part of the stress–strain curve for both the foam and the solid polyurethane allows a value of Young's modulus to be obtained.

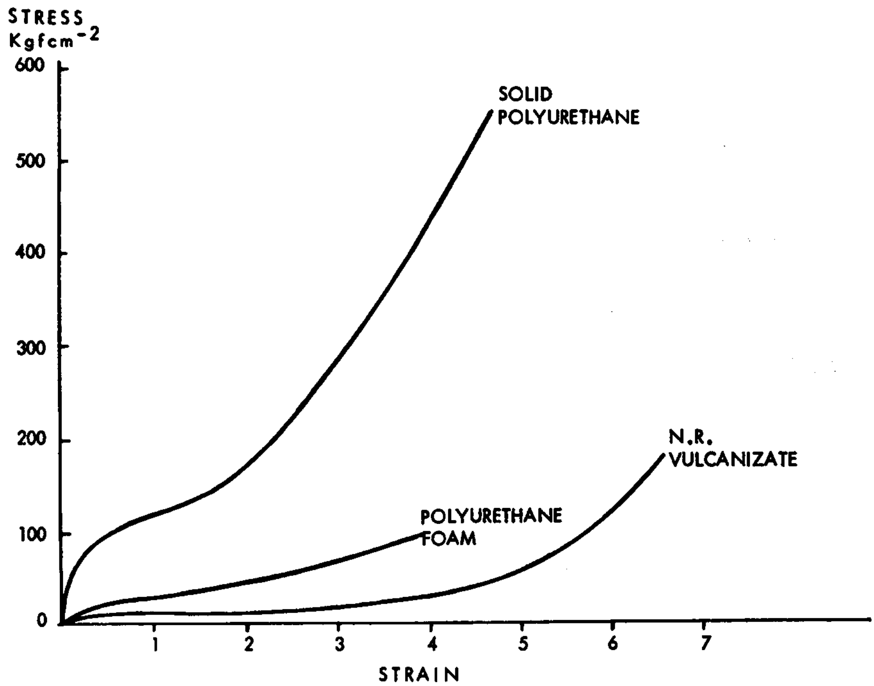

Fig. 4. Comparison of tensile stress–strain curves for polyurethane foam and solid and natural rubber vulcanizate.

It is possible to determine theoretically a value for Young's modulus of the foam, Y_F, by considering the extension of the model shown in Figure 2. If a small strain is applied parallel to one set of threads, Y_F can be obtained from the product of three factors: (i) Young's modulus of the solid material, Y; (ii) the strain magnification factor, eq. (1); (iii) factor representing the true load-bearing area, eq. (2), hence producing the equation

$$Y_F = Y \frac{\beta^2}{(1 + \beta)}. \tag{4}$$

Using a more complicated yet more realistic model of a system of n randomly disposed threads entering each intersection and approximating these by spheres of surface area nD^2, Gent and Thomas[11] found that the density of the foam was given by the same relation, eq. (3), and the equation for Young's modulus was only different by a factor of 2 from that given in eq. (4).

Hence, Young's modulus can be obtained from

$$Y_F = Y \frac{\beta^2}{2(1 + \beta)}. \tag{5}$$

The ratio Y_F/Y from the experimental results of Young's modulus for the two polyurethane foams and solid materials are plotted in Figure 5

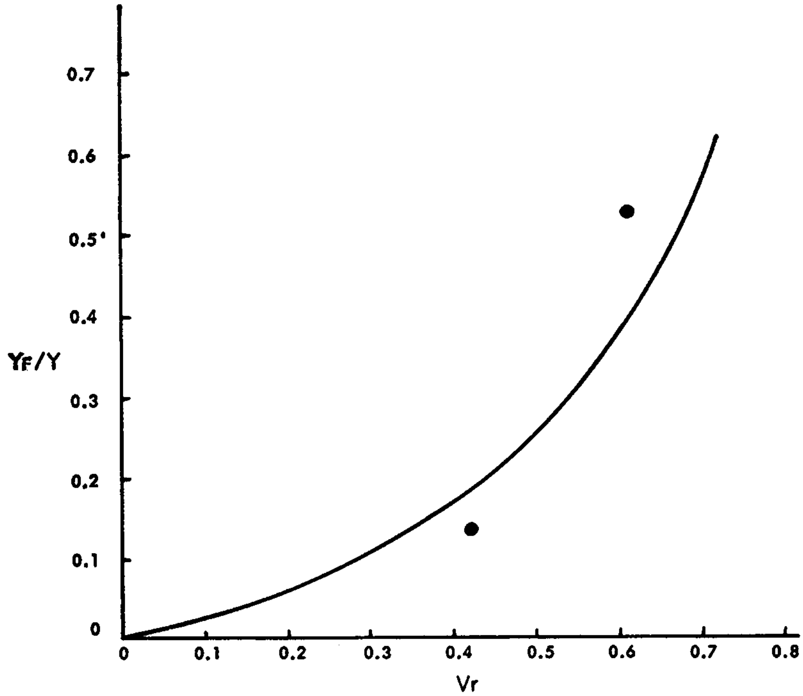

Fig. 5. Variation of Young's modulus of foam, Y_F, referred to that of the solid polyurethane, Y, with volume fraction, V_r, of rubber in the foam. Solid line is that predicted by eq. (5) from theory of Gent and Thomas.[13]

against the volume rubber fraction V_r determined from measured densities on the materials. Also shown on Figure 5 is the theoretical line obtained from eqs. (3) and (5). The values obtained for Young's modulus are therefore in reasonable agreement with theory. There is likely to be some error in the measurement of Young's modulus of the solid polymer in view of the difficulties involved in obtaining the material.

TEAR PROPERTIES

The most convenient method of measuring tear properties of rubber-like materials is to use the "tearing energy" approach developed by Rivilin and Thomas[21] from the classical theory on the strength properties of glass developed by Griffiths[22] in 1920. Tearing energy T is defined for a strained test piece containing a crack as

$$T = -\frac{\partial U}{\partial A} \tag{6}$$

where U is the total elastically stored energy in the test piece and A is the area of the cut surface. The derivative must be taken under conditions that the applied forces do not move and hence do not work. It thus repre-

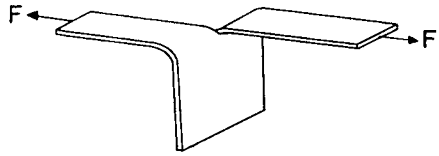

Fig. 6. "Trouser" tear test piece used in tearing energy measurments.

sents the rate of release of strain energy as the crack propagates and can therefore be considered as the energy available to drive the crack through the material. It has been found that if tear or crack-growth measurements are expressed in terms of T, the results obtained from test pieces of different shapes are the same, and hence values of T are characteristic of the material and not of the form of the test piece.[23] The "trouser" tear test piece shown in Figure 6 was used for the present investigation, as the value of T can readily be calculated from the applied force F by the relationship[21,24]

$$T = \frac{2F}{h} \tag{7}$$

where h is the test piece thickness.

Measured values of tearing energy from both foams are shown in Table I. A considerable difference was noted between the values of initial tearing and those for steady propagation of the tear. A similar difference in tearing energy values was also reported for latex foam rubbers by Gent and Thomas.[13]

TABLE I
Values of Tearing Energy

Foam	T (initiation), kg/cm	T (steady), kg/cm	β
1	12.7	43.5	0.816
2	20.3	54.7	1.35

The minimum theoretical value of tearing energy, T_F, for the model foam shown in Figure 2 is given by the energy required to break all the threads crossing a plane of unit area. The proportion of these threads to the total area of the foam structure is given by eq. (2), and hence the tearing energy of the foam is given by

$$T_F = E_r l_0 \frac{\beta^2}{(1 + \beta)^2} \tag{8}$$

where E_r is the breaking energy per unit volume of the bulk materials. The quantity l_0 (i.e., one thread length) is assumed in the theory to be the effective "width" of the tear tip and is obviously the minimum possible value. Assuming that the model shown in Figure 2 can be applied to poly-

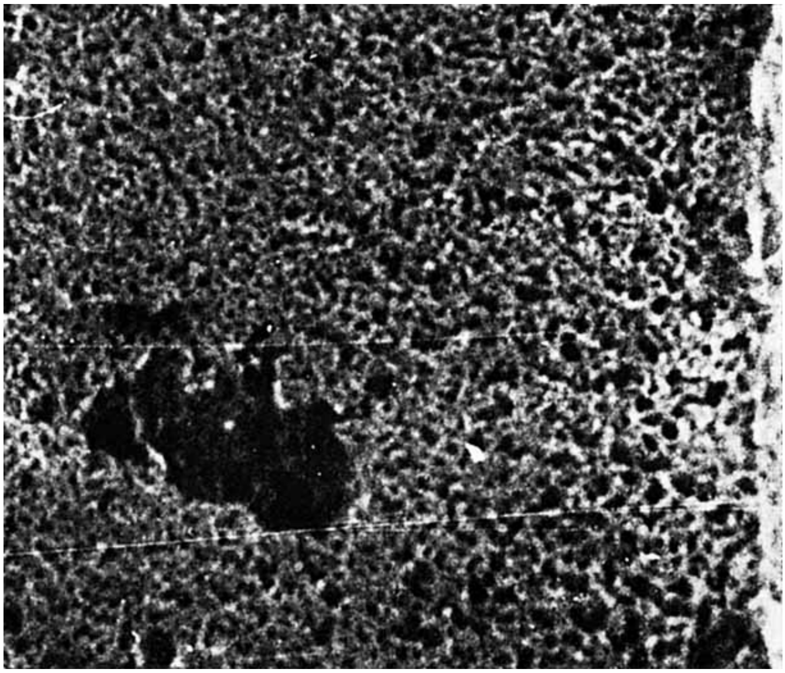

Fig. 7. Scanning electron microscope photograph of polyurethane foams used in this investigation showing that some pores can be of the order of 10^{-2} cms. Magnification 100×.

urethane foams, it is possible to calculate l_0 and compare this with the average and largest pore diameter obtained from microscopic measurements. The value β was found from the curve shown in Figure 3 by measuring the densities of the solid and foam polyurethanes. Values of β for foams 1 and 2 are listed in Table I. The values for E_r were found by graphically integrating the stress–strain curves for the two samples of solid polyurethane. On substituting these values in eq. (8), l_0 was found to be 4×10^{-2} cms for both foam 1 and foam 2. Although the average pore diameter is about 2×10^{-3} cm for both foams, odd pores as shown in the scanning electron microscope photograph in Figure 7 can be up to 2×10^{-2} cm in diameter.

Hence it can be considered that values for initiation of a tear can be obtained from eq. (8) by assuming that the effective width of the tear tip is about two times the largest pore diameter. This difference is probably due to imperfections in the foam causing local deviations of the tear from a linear path which gives rise to a corresponding larger effective tear width.

Gent and Thomas[12] found that the effective width of the tear tip for natural rubber foams at similar densities to the polyurethane foams in this paper was about five times the average pore diameter. Average pore diameters in their case, however, were a factor of 10 larger than those of the polyurethane foams used in this study.

TENSILE FAILURE

Following the tearing energy criterion developed by Rivlin and Thomas,[21] it can be assumed that tensile rupture occurs by catastrophic tearing from a flaw in one of the test piece surfaces. The tearing energy of the foam, T_F can then be expressed as[23]

$$T_F = 2kE_FL \tag{9}$$

for a test piece strained in simple extension where E_F is the energy density at failure in the bulk of the test piece for the foam, L is the depth of the flaw, and k is a numerical constant which varies slightly with strain[25] but can be taken for the purposes of this paper as having a value of 2.

The depth of flaw can then be calculated by measuring the tear strength and energy density to failure of the foam and substituting these values in eq. (10):

$$L = \frac{T_F}{4E_F} \tag{10}$$

The E_F value was obtained by measuring the area under the stress–strain curve of the foam. Using values of tear strength at initiation listed in Table I, L was found to be 1.72×10^{-2} cm for foam 1 and 2.3×10^{-2} cm for foam 2. These values are very close to the largest pore diameters measured from scanning electron microscopy photographs. The numerical agreement suggests that tensile failure occurs by catastrophic tearing from a flaw of the order of the largest pore in length. This conclusion is in agreement with the work by Gent and Thomas on natural rubber foams and hence accounts for the relatively low values of tensile strengths found in foam materials in general.

COMPRESSION

The type of stress–strain curve obtained in compression for the polyurethane foams used in poromerics is shown in Figure 8. Similar results have been reported previously for compression of polyurethane foams.[26,27] The type of curves obtained resemble those for the classical treatment of the buckling of a simple strut in compression. For foam 1, the ratio of thread length to width of the threads (i.e., β^{-1}) is 1.23, and hence the amount of buckling of the threads would be minimal. The classical Euler theory for buckling of struts can only be applied[28] if the length of the struts is at least 3.8 times their thickness, but it is informative to ascertain whether the assumed point of buckling (i.e., point at which curve changes slope) can be correlated with accepted "shape factor" theories of buckling from rubber engineering.

The critical compressive strain, e_c, of the individual threads of the model is given by[29]

$$e_c = \frac{\sigma_c}{Y(1 + 2S^2)} \tag{11}$$

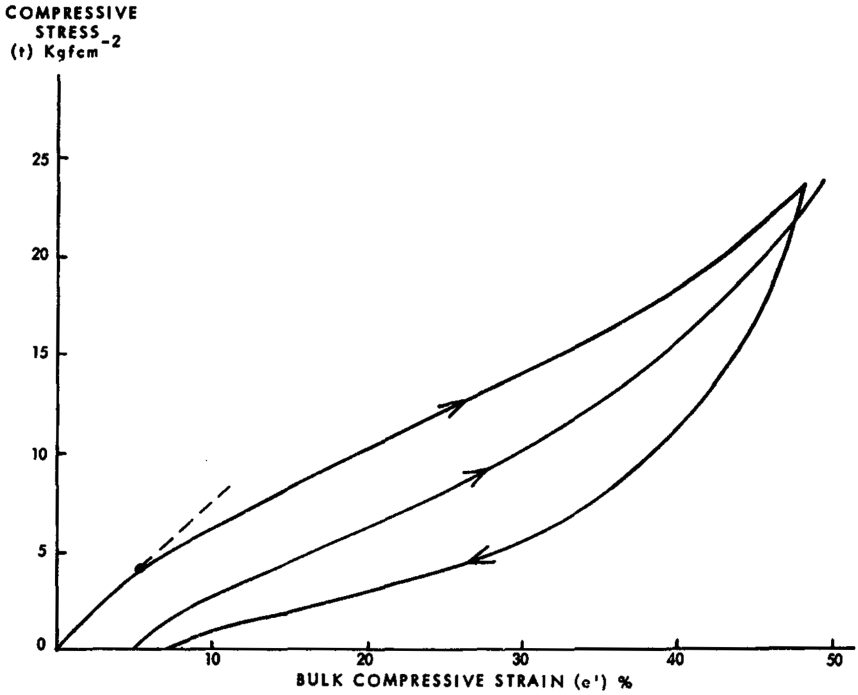

Fig. 8. Compressive stress–strain obtained for foam 1. Figure also shows retraction curve and second compressive stress–strain curve, indicating the large amount of stress softening and hysteresis.

where σ_c is the critical compressive stress, Y is Young's modulus of the solid material, and S is the shape factor of the strut in compression as discussed by Payne[30] and others[29] in rubber engineering theory and defined as the ratio of the one loaded area to the total force-free area, and given by

$$S = \frac{D^2}{4D\, l_0} = \frac{\beta}{4} \tag{12}$$

for a single rectangular strut such as those comprising the model structure shown in Figure 2.

For foam 1, S therefore has a value of 0.204. The value of stress at which the compression stress–strain curve in Figure 8 shows departure from linearity is 4.05 kgf/cm^2. The effective stress, however, on each strut in the model will be much higher, as they only occupy a fractional area of the total as given by eq. (2). For β of 0.816, as in the case with foam 1, the threads occupy only 0.2 of the total cross-sectional area of the foam; hence the critical buckling stress σ_c for each thread in the model is 20.25 kgf/cm^2.

Using this value for σ_c and the value of 387 kgf/cm^2 predicted by eq. (5) for Young's modulus of the solid material from measurements of Young's modulus on the foam, a value for critical compressive strain e_c of the struts of 0.05 is obtained. The actual effective buckling strain of the foam,

e'_c, will be lower, however, due to the undeformable regions at thread intersections as predicted by eq. (1). Hence the critical compressive strain of the foam from theory is 0.03, which compares reasonably well with the experimental value of 0.056 obtained from the compression stress strain curve in Figure 8.

An alternative approach adopted by Gent and Thomas[13] is to include in the classical Euler strut theory an unknown function of strain, $f(e)$. The compression is assumed to be directed parallel to one set of threads in the model structure shown in Figure 2 and to take place by buckling of these threads. The force F on each thread is given by

$$F = \frac{YAK^2 f(e)}{l_0^2} \tag{13}$$

where AK^2 is the moment of inertia of the thread cross section. For threads of similar cross section, $AK^2 = mD^4$, where m is a constant. The number of threads per unit cross-sectional area is given by $(l_0 + D)^{-2}$, and hence the average compressive stress t is given by

$$t = \frac{F}{(l_0 + D)^2} = Yf(e)\,\frac{\beta^4}{(1 + \beta)^2} \tag{14}$$

by substituting β for D/l_0 and absorbing the constant m in $f(e)$.

Bulk compressive strain e' is, however, influenced by two factors which can be considered additive: firstly the incompressibility of thread intersections as predicted by eq. (1) and secondly a contribution from simple compression of the threads by an amount t/Y_F. Hence the bulk compressive strain e' will be given by

$$e' = \frac{e}{1 + \beta} + \frac{t}{Y_F}. \tag{15}$$

The application of such a theory to polyurethane foams is difficult, for, as can be seen in Figure 8, they display a large amount of energy loss or hysteresis and also a considerable amount of stress softening (i.e., the reduction in stress on the second extension curve). Although stress softening has been extensively studied[19,31] in tension, little work appears to have been reported on stress-softening effects in compression.

Despite the large amount of hysteresis, the analysis along the lines suggested by Gent and Thomas has, however, been also adopted in this paper to ascertain the form of the function $f(e)$.

By substituting measured values for bulk compressive stress t and bulk compressive strain e' from Figure 8, it is possible, using the derived value for β and experimental values of Young's modulus for the solid polyurethane and foam, to obtain corresponding values of $f(e)$ and effective strain e of the threads in the model by use of eqs. (14) and (15). The relationship derived is shown in Figure 9. It is of the general form expected for a buckling process and is very similar to that obtained for natural rubber foams,[13] al-

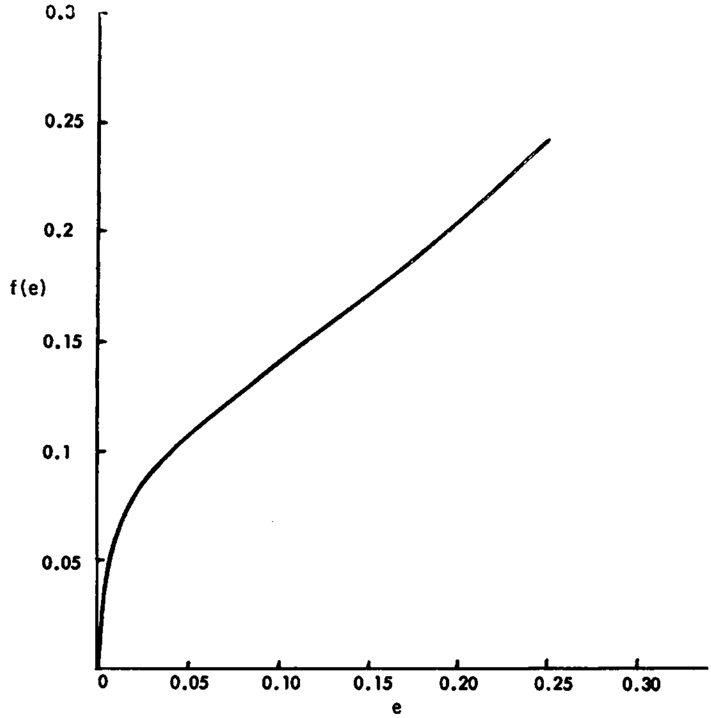

Fig. 9. Variation of $f(e)$ with thread compressive strain e obtained from stress-strain curve shown in Figure 8.

though compression tests on the latter were only reported for foams with values of V_r less than 0.2.

CONCLUSIONS

The theoretical model proposed by Gent and Thomas which has been applied in this paper to the mechanical behavior of polyurethane foams is a very idealized representation of an actual foam, which in practice must be far from homogeneous. The actual threads and intersections are of a wide range of shapes and sizes, as can be seen from the stereoscan photomicrographs. The apparent good agreement therefore obtained between experimental values of Young's modulus and theory is very satisfactory, particularly in view of the difficulties that occur with obtaining a reasonably good sample of solid material.

The measured values of breaking energy are in good agreement with those calculated on the assumption that tensile failure occurs by tearing at the tip of the largest pore, which is the same criterion as that found for natural rubber foams.

Values for tear strength at the initiation of a flaw can be obtained from the theory by assuming that the effective width of the tear tip is about twice the largest pore diameter. Tear strength results on polyurethane foams thus appear to differ from those on natural rubber foams as Gent and Thomas found that tear strength was much more dependent on the average pore diameter. In the case of the polyurethane foams examined in this paper the average pore diameter was at least a factor of 10 lower than the maximum pore diameter.

The shape of the compression stress–strain curve is similar to that obtained from the buckling of a strut in simple compression and can reasonably be described by the model proposed on the assumption that the threads in the model buckle under a compressive load. The arbitrary function $f(e)$ provides a measure of the inhomogeneity of the foam structure, and the variation of $f(e)$ with strain is of the same form as that found for natural rubber foams. An alternative approach by use of shape factor theories predicts within a factor of 2 the value of the compressive buckling strain as compared with the value shown by the deviation in linearity of the compression stress–strain curve.

Thus, the fairly simple model of a collection of thin threads of equal length joined together to form a cubical lattice appears to predict reasonably well the mechanical behavior of polyurethane foams used in poromerics.

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